1,25

D_5

"التفاضل الضمني"

Implicit Differentiation

التفاضل الضمني

Ex $y = \tan^{-1}x + (\sin x)^x$

هنا y داله في x صريحة ولذلك y' تصبح داله

في x فقط

وعكس لو $\Longleftarrow$ $y = y^2\tan^{-1}x + e^y + x^2$

هنا تكون y موجوده في أكثر من ترم ولا يمكن فصل y لوحده

ولذلك التفاضل هنا يسمى تفاضل ضمني حيث يتم التفاضل

وتجميع y' في طرف لوحده ولتصبح y' داله في $\boxed{x, y}$

Ex For $\Rightarrow$ $y^{10} = 3y^2 x + \cos x^2 + e^y$

Find $\frac{dy}{dx}$

Soln $10y^9 y' = 3y^2 + x\,6y\,y' - \sin x^2 (2x) + e^y y'$

$$y'\left[10y^9 - 6xy - e^y\right] = 3y^2 - 2x\sin x^2$$

$$\therefore y' = \frac{3y^2 - 2x\sin x^2}{10y^9 - 6xy - e^y}$$

Ex $\tan(xy) = e^{x/y} + 5$

Find $\frac{dy}{dx}$

Soln

$$\sec^2(xy)\left[y + xy'\right] = e^{x/y}\left[\frac{y - xy'}{y^2}\right] + \text{Zero}$$

$$y'\left[x\sec^2 xy + e^{x/y}\frac{x}{y^2}\right] = e^{x/y}\frac{1}{y} - y\sec^2(xy)$$

$$\therefore\ y' = \frac{\frac{1}{y}e^{x/y} - y\sec^2(xy)}{x\sec^2 xy + \frac{x}{y^2}e^{x/y}}$$

$\boxed{5}$ $$x^{y+1} + y^{x+1} = 1$$

<u>Soln</u>

$$x^{y+1} \ln x \; y' + (y+1)\, x^{y} \cdot 1$$

$$+ y^{x+1} \ln y \cdot 1 + (x+1)\, y^{x}\, y' = 0$$

$$y' \left[x^{y+1} \ln x + y^{x} (x+1) \right] = -x^{y}(y+1) - y^{x+1} \ln y$$

$$y' = - \frac{x^{y}(y+1) + y^{x+1} \ln y}{x^{y+1} \ln x + y^{x}(x+1)}$$

3] $\tan^{-1}\frac{y}{x} = \frac{1}{2}\ln(x^2+y^2)$

Soln $\frac{1}{1+\left(\frac{y}{x}\right)^2}\left(\frac{y'}{x}+\left(\frac{-y}{x^2}\right)\right) = \frac{1}{2}\;\frac{2x+2yy'}{x^2+y^2}$

$$\frac{y'}{1+(y/x)^2}\frac{1}{x} - \frac{y}{x^2+y^2} = \cancel{\frac{1}{2}}\left(\frac{\cancel{2}x}{x^2+y^2}+\frac{\cancel{2}yy'}{x^2+y^2}\right)$$

$$y'\left[\frac{1}{x}\frac{1}{1+(y/x)^2}-\frac{y}{x^2+y^2}\right] = \frac{x}{x^2+y^2}+\frac{y}{x^2+y^2}$$

$$y' = \frac{\frac{(x+y)}{x^2+y^2}}{\frac{1}{x+y^2/x} - y/x^2+y^2} \quad \#$$

Ex $y^2 + y\cos x = x^y$

Soln $2yy' + \cos x (y') - y\sin x$

$= x^y \ln x \, y' + y x^{y-1} \cdot 1$

$y'\left[2y + \cos x - x^y \ln x\right] = y\sin x + y\, x^{y-1}$

$$y' = \frac{y\sin x + x^{y-1}y}{2y + \cos x - x^y \ln x}$$

7

9] $\tanh y + 2^{\tanh^{-1}x} = \sinh xy$

Soln

$$\operatorname{sech}^2 y\,(y') + 2^{\tanh^{-1}x}\ln 2\,\frac{1}{1-x^2} = \cosh xy\,(xy' + y)$$

$$y'\left[\operatorname{sech}^2 y - x\cosh xy\right] = y\cosh xy - 2^{\tanh^{-1}x}\ln 2\,\frac{1}{1-x^2}$$

$$y' = \frac{y\cosh xy - 2^{\tanh^{-1}x}\ln 2\left(\frac{1}{1-x^2}\right)}{\operatorname{sech}^2 y - x\cosh xy}$$

$\boxed{8}$

$\boxed{6}$ $$(\sin x)^{\cos y} - y e^{\tan^{-1} x} = 10$$

Soln

$$(\sin x)^{\cos y} \ln(\sin x)\,(-\sin y)\, y'$$

$$+ (\cos y)(\sin x)^{\cos y - 1} \cos x - e^{\tan^{-1} x}\, y'$$

$$- y e^{\tan^{-1} x} \frac{1}{1+x^2} = 0$$

$$y'\left[-\sin y(\sin x)^{\cos y} \ln(\sin x) - e^{\tan^{-1} x}\right]$$

$$= y e^{\tan^{-1} x} \frac{1}{x^2+1} - (\cos y)(\sin x)^{\cos y - 1} \cos x$$

$$y' = \frac{y e^{\tan^{-1} x}\left(\frac{1}{x^2+1}\right) - (\cos y)(\sin x)^{\cos y - 1} \cos x}{-\sin y\,(\sin x)^{\cos y} \ln \sin x - e^{\tan^{-1} x}}$$

3] $\cos^{-1}\left(\frac{x}{y}\right) = \sin(x^2 y) + 10$

Soln $\frac{-1}{\sqrt{1-\left(\frac{x}{y}\right)^2}}\left[\frac{1}{y} + \left(\frac{-x}{y^2}\right)y'\right]$

$= \cos(x^2 y)\left[2xy + x^2 y'\right]$

$y'\left[\frac{x}{y^2}\frac{1}{\sqrt{1-(x/y)^2}} - x^2\cos(x^2 y)\right] = 2xy\cos(x^2 y) + \frac{1}{y}\frac{1}{\sqrt{1-(x/y)^2}}$

$\therefore y' = \checkmark$

] $y^2 + y\cos x + 3^{x^2} = x^y$

Soln $2yy' + y'\cos x - y\sin x + 3^{x^2}(2x)\ln 3 =$

$y x^{y-1} + x^y \ln x \, y'$

$y' = \frac{y x^{y-1} + y\sin x - 3^x \ln 3 (2x)}{(2y + \cos x - x^y \ln x)}$

10

[7] $(\sec xy)^{x} + \ln(\tan^{-1}x)^{y} = 10$

Soln

$(\sec xy)^{x} + y\ln(\tan^{-1}x)$

$(\sec xy)^{x}\ln(\sec xy)\,1 + x(\sec xy)^{x-1}(\sec xy)(\tan xy)$

$[xy' + y] + y'\ln\tan^{-1}x + y\dfrac{1}{\tan^{-1}x}\dfrac{1}{x^{2}+1} = 0$

$y'\left[x^{2}(\sec xy)^{x-1}\sec(xy)\tan(xy) + \ln\tan^{-1}x\right]$

$= \left[-(\sec xy)^{x}\ln\sec xy - \dfrac{y}{\tan^{-1}x}\dfrac{1}{x^{2}+1}\right]$

$y' =$ ___